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An ORESTE Approach based Decision-Making Framework for Renewable Energy Development with Probabilistic Dual Hesitant Fuzzy Information

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Abstract

Development and utilization of renewable energy sources plays a crucial role in the sustainable development of a country. Selection among energy sources is a Multi-Criteria Decision-Making (MCDM) problem. Therefore, it is necessary to make an assessment in terms of several conflicting criteria. The aim of this paper is to develop the French Organization Rangement Et Synthese De Ronnees Relationnelles' (ORESTE) approach for ranking the renewable energy plant under probabilistic dual hesitant fuzzy environment. First, an algorithm has been developed to normalize the Probabilistic Dual Hesitant Fuzzy Element (PDHFE). A series of new distance and similarity measures for PDHFEs under both discrete and continuous environments has also been developed in this article. To illustrate how well the developed method works, a sustainable renewable energy development problem has been taken to rank the renewable energy plant. To validate the working of the developed method a Sustainable Supplier Selection (SSS) problem has also been taken. To verify the robustness of proposed methodology, a comparative study and sensitive analysis has also been discussed.

Keywords: Probabilistic dual hesitant fuzzy sets, ORESTE, Multi-criteria decision making, renewable energy development, Supply chain management.

1 | Introduction

In the era of globalization, energy planning is one of the important concerns for many countries. The growing population, urbanization, and industrialization gradually increase the demand for natural resources and

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energy. With the increasing use of non-renewable energy resources, global warming, and climate change are becoming one of the most difficult problems faced by many developing countries. In order to address the environmental issues coming in the path of sustainable development, renewable energy sources can play a crucial role. Therefore, renewable energy sources [1] can prove very beneficial to tackling climate change and for the sustainable development of the country.

For the sake of sustainable development, Rasul developed a Sustainable Supply Chain Management (SSCM) in 2016 [2], which includes every aspect of sustainability, including its economic, social, and environmental dimensions. Any company's supply chain needs to adjust to the market conditions. Energy consumption, waste production, air pollution, water pollution, climate change, and safety are some significant issues in the supply chain. The supplier selection process [3] is the most important component of SSCM and is based on Multi-Criteria Decision-Making (MCDM). Decision-making aims to choose the optimal alternative from various feasible alternatives depending on different criteria. Since the parameters and information related to real-life problems include uncertainty. These problems are resolved by using different MCDM techniques. Several uncertainty sets have been proposed for rational decision-making that give reliable results.

Fuzzy numbers are one of the well-known and often-used approaches to address these uncertainties. Zadeh [4] gave the idea of fuzzy sets. Many researchers have been interested in Fuzzy sets, and these sets are widely applicable in a number of directions, such as industry [5], academia [6], [7], and supply chain management [8]. Fuzzy sets have various applications, but they are unable to address the falsity and degree of indeterminacy of context. Therefore, to fill these gaps, some crucial extensions have been carried out, such as Intuitionistic Fuzzy Sets (IFS) [9], interval-valued IFSs [10–12], Hesitant Fuzzy Sets (HFS) [13], the Dual Hesitant Fuzzy Sets (DHFS) [14] Although these extensions are capable of capturing uncertainties efficiently. Further, if Decision-Makers (DMs) are leaning towards an element in their evaluation, then HFSs or DHFSs are unable to represent such an evaluation.

This generates a huge loss of knowledge and may lead to unsatisfactory outcomes. To overcome such a situation, Zhu and Xu [15] give the idea of Probabilistic Hesitant Fuzzy Sets (PHFSs). Further, Hao [16] introduced Probabilistic Dual Hesitant Fuzzy Sets (PDHFSs), which allow the DMs to provide fuzzy membership and non-membership based on probabilistic information.

MCDM is a versatile technique that helps DMs examine problems and make decisions by simultaneously considering all the alternatives, criteria, and preferences of DMs. Several steps have been required to solve an MCDM problem, such as normalization, aggregation of data, and ranking of alternatives. Outranking-based techniques and utility value-based methods are two major types of ranking techniques. The outranking-based techniques such as QUALIFLEX [17], ELECTRE [18], [19] and others [20–22] are based on pairwise comparisons of alternatives. These methods are able to determine the preference, indifference, and incomparability relations among the alternatives. Although DM determines the threshold to differentiate these relationships, the results obtained are not reliable as they depend upon DMs' preference.

Utility-based approaches rank alternatives by aggregating the cost of each alternative based on all criteria. Some majorly used utility-based methods are VIKOR [23] and TOPSIS [24]. Although the results obtained by these techniques are more instinctive and clearer, these techniques failed to give an incompatibility relationship between the two choices. To deal with the incomparability of two alternatives, the French Organization Management Et Synthese De Donnees Relationnelles' (ORESTE) [25], [26] method was introduced by Roubens in 1982. In the ORESTE method, the preference of the criteria and alternatives under each criterion can determine the preference structure. Recently, it has been emerging in several fuzzy environments, such as hesitant fuzzy [27], [28], intuitionistic fuzzy [29], and probabilistic fuzzy environment [30].

Since utility-based methods like VIKOR and TOPSIS have a number of advantages in ranking the alternatives, they are still unable to solve the aforementioned shortcomings. To overcome these shortcomings, the Probabilistic Hesitant Fuzzy (PHF) ORESTE method is developed by [25], which shows the

incomparability of the relationship between the alternatives. Although the existing ORESTE methods are efficient in solving MCDM problems, they are only defined for PHF information and have some limitations. Literature has insisted on using a normalized-based method to aggregate DMs' opinions. The limitation of existing methods, with respect to DM's risk preference, is that different forms of normalized fuzzy elements are formed. This results in different distance measure values. Keeping in mind these problems and integrating the information provided by the DMs, there is a need to develop a normalization method that can give a single normalized fuzzy element irrespective of DM's preference.

In the available literature, no study has been found in which the ORESTE method is defined for PDHF information. Motivated by this, we have defined the ORESTE approach for PDHF information in this study. The purpose of this study is to introduce a novel normalization approach to Normalize Probabilistic Dual Hesitant Fuzzy Elements (NPDHFEs). A series of novel distance and similarity measures have also been defined to find the distance between two NPDHFEs. Afterwards, the ORESTE approach is extended to probabilistic dual hesitant fuzzy environments.

The main contributions of this study are summarised as follows:

I. A novel normalization approach to normalize NPDHFEs has been developed.

II. A series of novel distance and similarity measures for PDHFS have been defined under both continuous and discrete environments.

III. An algorithm for the probabilistic dual hesitant fuzzy ORESTE method has been developed.

IV. An illustrative example of renewable energy development has been taken to illustrate the developed ORESTE model.

V. A comparative analysis along with a sensitive analysis has also been conducted with the existing problem in the field of sustainable suppliers' selection under a probabilistic dual hesitant fuzzy environment to validate the applicability of the proposed method.

The rest of this paper is organized as follows. In Section 2, the basic definition of DHFS and PDHFS has been given. Section 3 involves a new normalization process to obtain NPDHFEs and a series of new distance and similarity measures. Afterward, the PDHFE-ORESTE approach has been defined in Section 4. In Section 5, the application of the produced method in the field of renewable energy development is illustrated. Also, to validate the workings of the proposed method, an existing problem in the field of sustainable suppliers' selection under the PDHF environment has been discussed, along with a comparative study and sensitive analysis. Section 6 contains the concluding remark.

2 | Preliminaries

In this section, some fundamental definitions of DHFS and PDHFS are discussed.

2.1 | Dual Hesitant Fuzzy Sets

Definition 1 ([14]). Let X be the set of discourse, then the DHFS D on X is defined as

$$D = \{\langle x, h(x), g(x)\rangle | x\epsilon X\}, \quad \textbf{(1)}$$

Where $h(x)$ and $g\,(x)$ represent the sets of possible Membership Degree (MD) and Non-Membership Degree (NMD) of the element $x\epsilon X$, to the set D, respectively, such that $\gamma(x) \in h(x)$, $\eta(x) \in g(x)$ where $\gamma(x) \in [0,1], \eta(x) \in [0,1]$ and , $0 \leq \gamma^{+}(x) + \eta^{+}(x) \leq 1$, where $\gamma^{+}(x) = \max \gamma(x)$ and $\eta^{+}(x) = \max \eta(x)$ for all $x\epsilon X$. In particular, the pair $\{\langle h(x), g(x)\rangle\}$ is called a Dual Hesitant Fuzzy Element (DHFE), which can be denoted as $\{\langle h, g\rangle\}$.

2.2 | Probabilistic Dual Hesitant Fuzzy Sets

Definition 2 ([16]). Let X be a reference set; a PDHFS on X is defined by

$$P = \{\langle x, h_x|(p_x), g_x|(q_x)\rangle | x \epsilon X\}. \tag{2}$$

where h_x and g_x represent the set of MD and NMD to the set X of x and p_x and q_x are corresponding probabilistic information of these two types of degrees.

Also, there is

$$0 \le \gamma, \eta \le 1; 0 \le \gamma^+ + \eta^+ \le 1. \tag{3}$$

and

$$p_i \epsilon [0,1], q_j \epsilon [0,1], \sum_{i=1}^{\#h} p_i = 1, \sum_{j=1}^{\#g} q_j = 1, \tag{4}$$

Where,

$$\gamma \in h(x), \eta \in g(x), \gamma^+ \in h^+ (x) = \bigcup_{\gamma \in h(x)} \max\{\gamma\} \text{ and } \eta^+ \in g^+ (x) = \bigcup_{\eta \epsilon g(x)} \max\{\eta\}. \tag{5}$$

and $p_i \epsilon p_x$ and $q_j \epsilon q_x$. $\#h$ and $\#g$ are the total number of elements in $h_x(p_x)$ and $g_x(p_x)$ respectively. For convenience, we shall name $(h_x(p_x), g_x(q_x))$ as PDHFE.

3 | Novel Normalization Approach and Series of Distance and Similarity Measures for NPDHFEs

In this section, we have discussed the limitations of the existing algorithm used to obtain the NPDHFEs. Moreover, an algorithm to obtain new NPDHFEs has been developed to overcome the drawbacks of existing methods. A series of new distance and similarity measures have also been developed in this section.

3.1 | An Approach to Finding Normalized NPDHFEs

From the literature, we can conclude that most of the measures, such as distance and similarity measures, are based on the assumption that the length of possible MD and NMD in two PDHFSs are equal. To achieve this, several techniques can be used to unify the length. To achieve this, under a probabilistic dual hesitant fuzzy environment, the experts need to add the least or the largest value among the set of MD and NMD of the smaller set until the length of both PDHFSs becomes equal [25].

The choice of value's repetition entirely depends on the optimistic or pessimistic nature of DM. If the DM is optimistic, then the largest value of membership and the smallest value of non-membership will be added to the MD and NMD sets of a smaller set. However, if DM follows a pessimistic approach, then the least value of membership and the largest value of non-membership will be added to set the MD and NMD in a smaller set. Obviously, the original information of PDHFSs has been changed when we normalize the length of PDHFEs by existing methods [25]. This results in the distortion of evaluation results. In the following subsection, we discuss the drawbacks of the existing normalization procedures and define a novel normalization approach to obtain NPDHFEs.

Drawbacks of existing algorithm to normalize NPDHFEs

In this subsection, the limitations of existing NPDHFEs are discussed and presented in [25]. An algorithm has also been designed to obtain new NPDHFEs.

In order to understand the existing normalized approach [25], suppose an element has to be added to PDHFE $h(p) = \langle \begin{Bmatrix} 0.4|0.2, 0.3|0.1 \\ 0.2|0.2, 0.3|0.5 \end{Bmatrix}, \{0.4|0.3, 0.5|0.7\} \rangle$ to obtain NPDHFE.

Case 1. If the DMs are pessimistic, then the lowest MD 0.2 is added to the membership set, and the maximum NMD 0.5 will be added to non-membership set of $h(p)$. Therefore, the NPDHFE will become

$$h^N(p^N) = \langle \begin{Bmatrix} 0.4|0.2, 0.3|0.1 \\ 0.2|0.2, 0.2|0, 0.3|0.5 \end{Bmatrix}, \{0.4|0.3, 0.5|0.7, 0.5|0\} \rangle.$$

Case 2. If the DMs are optimistic, then the highest MD 0.4 is added to the membership set, and the minimum NMD 0.4 will be added to non-membership set of $h(p)$. Therefore, the NPDHFE will become

$$h^N(p^N) = \langle \begin{Bmatrix} 0.4|0.2, 0.3|0.1 \\ 0.2|0.2, 0.4|0, 0.3|0.5 \end{Bmatrix}, \{0.4|0.3, 0.4|0, 0.5|0.7, \} \rangle.$$

Clearly, a different form of NPDHFEs has been obtained with respect to the nature of DMs with diverse risk choices. Therefore, the normalization process is influenced by the nature of DMs. Depending upon these NPDHFEs, different distance measures will be obtained.

Hence, an issue arises on obtaining the NPDHFEs that are not influenced by DMs preferences. To overcome this issue, a new algorithm has been defined to obtain NPDHFEs by considering the normalized approach developed in [31].

Novel algorithm to normalize NPDHFEs

Jian [31] developed a normalized approach to obtain NPHFEs. By considering this approach, the following algorithm has been designed to obtain NPDHFEs. This section contains a novel algorithm to obtain the NPDHFEs with the same set of probabilities.

Let for $x \epsilon X$, A and B be two PDHFE given by $A = \{\langle x, h_1(p_1), g_1(q_1)\rangle\}$ and $B = \{\langle x, h_2(p_2), g_2(q_2)\rangle\}$, where $h_1(p_1) = \{\gamma_1^{m_1}(p_1^{m_1}) | m_1 = 1,2, \dots, \#h_1\}$ and $g_1(q_1) = \{\eta_1^{n_1}(q_1^{n_1}) | n_1 = 1,2, \dots, \#g_1\}$ are sets of MD and NMD for the element $x \epsilon X$ for set A. Similarly, $h_2(p_2) = \{\gamma_2^{m_2}(p_2^{m_2}) | m_2 = 1,2, \dots, \#h_2\}$ and $g_2(q_2) = \{\eta_2^{n_2}(q_2^{n_2}) | n_2 = 1,2, \dots, \#g_2\}$ be set of MD and set of NMD for the element $x \epsilon X$ for set B.

Algorithm 1. Input: for two sets of membership of two PDHFEs $h_1(p_1)$ and $h_2(p_2)$ where $h_1(p_1) = \{\gamma_1^{m_1}(p_1^{m_1}) | m_1 = 1,2, \dots, \#h_1\}$ and $h_2(p_2) = \{\gamma_2^{m_2}(p_2^{m_2}) | m_2 = 1,2, \dots, \#h_2\}$.

Output: two NPDHFEs $h_1^N(p_1^N)$ and $h_2^N(p_2^N)$, where $h_1^N(p_1^N) = \{\gamma_1^{m_1}(p_1^{m_1}) | m_1 = 1,2, \dots, \#h_1\}$ and $h_2^N(p_2^N) = \{\gamma_2^{m_2}(p_2^{m_2}) | m_2 = 1,2, \dots, \#h_2\}$.

Step 1. Determine the first element $h_1^N(p_1^N)$ and $h_2^N(p_2^N)$. If $(p_1^1) \geq (p_2^1)$, then $(\gamma_1^1(p_1^1) = \gamma_1^1(p_2^1)$ and $(\gamma_2^1(p_2^1) = \gamma_2^1(p_2^1)$. Otherwise, $\gamma_1^1(p_1^1) = \gamma_1^1(p_1^1)$ and $\gamma_2^1(p_2^1) = \gamma_2^1(p_1^1)$.

Step 2. Determine the second element $h_1^N(p_1^N)$ and $h_2^N(p_2^N)$. If $(p_1^1) \geq (p_2^1)$ and $(p_1^1) - (p_2^1) \leq (p_2^2)$, then $\gamma_1^2(p_1^2) = \gamma_1^1((p_1^1) - (p_2^1))$ and $(\gamma_2^2(p_2^2) = \gamma_2^2((p_1^1) - (p_2^1))$. If $(p_1^1) \geq (p_2^1)$ and $(p_1^1) - (p_2^1) > (p_2^2)$, then $\gamma_1^2(p_1^2) = \gamma_1^1(p_2^2)$, and $(\gamma_2^2(p_2^2) = \gamma_2^2(p_2^2)$. If $(p_1^1) < (p_2^1)$ and $(p_2^1) - (p_1^1) \leq (p_1^2)$, then $\gamma_1^2(p_1^2) = \gamma_1^2((p_2^1) - (p_1^1))$ and $\gamma_2^2(p_2^2) = \gamma_2^1((p_2^1) - (p_1^1))$. If $(p_1^1) \leq (p_2^1)$ and $(p_2^1) - (p_1^1) > (p_1^2)$, then $\gamma_1^2(p_1^2) = \gamma_1^2(p_1^2)$ and $\gamma_2^2(p_2^2) = \gamma_2^1(p_1^2)$.

Step 3. Determine the third element $h_1^N(p_1^N)$ and $h_2^N(p_2^N)$. If $(p_1^1) \geq (p_2^1)$, $(p_1^1) - (p_2^1) \leq (p_2^2)$ and $(p_1^2) \leq (p_2^2) - (p_1^1) + (p_2^1)$, then $\gamma_1^3(p_1^3) = \gamma_1^2(p_1^2)$ and $\gamma_2^3(p_2^3) = \gamma_2^2(p_1^2)$. If $(p_1^1) \geq (p_2^1)$, $(p_1^1) - (p_2^1) \leq (p_2^2)$ and $(p_1^2) > (p_2^2) - (p_1^1) + (p_2^1)$, then $\gamma_1^3(p_1^3) = \gamma_1^2(p_2^2 + p_2^1 - p_1^1)$ and $\gamma_2^3(p_2^3) = \gamma_2^2(p_2^2 + p_2^1 - p_1^1)$. If $p_1^1 \geq p_2^1, p_1^1 - p_2^1 > p_2^2$ and $p_1^2 - p_2^2 < p_2^3$, then $\gamma_1^3(p_1^3) = \gamma_1^2(p_1^2 - p_2^2)$ and $\gamma_2^3(p_2^3) = \gamma_2^3(p_1^2 - p_2^2)$. If $p_1^1 < p_2^1$, $p_2^1 - p_1^1 \leq p_1^2$ and $p_1^2 - p_2^1 + p_1^1 \leq p_2^2$ then $\gamma_1^3(p_1^3) = \gamma_1^2(p_1^2 - p_2^1 + p_1^1)$ and $\gamma_2^3(p_2^3) = \gamma_2^2(p_1^2 - p_2^1 + p_1^1)$. If $p_1^1 < p_2^1, p_2^1 - p_1^1 > p_1^2$ and $p_2^1 - p_1^1 + p_1^2 \leq p_1^3$, then $\gamma_1^3(p_1^3) = \gamma_1^3(p_2^1 - p_1^1 - p_1^2)$ and $\gamma_2^3(p_2^3) = \gamma_2^1(p_2^1 - p_1^1 - p_1^2)$. If $p_1^1 < p_2^1, p_2^1 - p_1^1 > p_1^2$ and $p_2^1 - p_1^1 + p_1^2 > p_1^3$, then $\gamma_1^3(p_1^3) = \gamma_1^3(p_1^3)$ and $\gamma_2^3\ (p_2^3) = \gamma_2^1(p_1^3)$.

Similarly, the values $\gamma_1^{m_1}(p_1^{m_1})$ and $\gamma_2^{m_2}(p_2^{m_2})$, $(m_1 = 4,5, \dots, h_1\ )$, $(m_2 = 4,5, \dots, h_2\ )$ can be determined. When $p_1^1 + p_1^2 + \cdots + p_1^{\#h_1} = 1$ and $p_2^1 + p_2^2 \dots + p_2^{\#h_2} = 1$ then $h_1^N(p_1^N)$ and $h_2^N(p_2^N)$ have been derived.

Step 4. End.

Similarly,

Input: for two sets of NMDs of two PDHFEs $g_1(q_1)$ and $g_2(q_2)$ where $g_1(q_1) = \{\eta_1^{n_1}(q_1^{n_1})|n_1 = 1,2,\dots,\#g_1\}$ and $g_2(q_2) = \{\eta_2^{n_2}(q_2^{n_2})|n_2 = 1,2,\dots,\#g_2\}$

Output: two NPDHFEs $g_1^N(g_1^N)$ and $g_2^N(p_2^N)$, where $g_1^N(p_1^N) = \{\eta_1^{n_1}(q_1^{n_1})|n_1 = 1,2,\dots,\#g_1\}$ and $g_2^N(q_2^N) = \{\eta_2^{n_2}(q_2^{n_2})|n_2 = 1,2,\dots,\#g_2\}$.

Proceed with *Algorithm 1*.

Remark 1. For n steps, there are 2^{n+1} judgement conditions. For each judgement condition, we have the following relationship: $p_1^1 = p_2^1; p_1^2 = p_2^2; \dots p_1^{\#h_1} = p_2^{\#h_2}$. With the aid of a computer, these conditions can easily be recognized.

Remark 2. The formula $\gamma_1^1(p_1^1) = \gamma_1^1(p_2^1)$ is equivalent to $\gamma_1^1(p_1^1) = \gamma_1^1(p_1^1)$ and $p_1^1 = p_2^1$ in *Step 1*. Similarly, further formulas presented in *Step 2* and *Step 3* can be explained.

Remark 3. The relationship$h_1^N(p_1^N) = h_1(p_1)$ and $h_1^N(p_1^N) = h_2(p_2)$ in the algorithm may not be true when $h_1(p_1)$ and $h_2(p_2)$ have the same number of elements.

Remark 4. For input PDHFEs $h_1(p_1)$ and $h_2(p_2)$ we have $\sum_{i=1}^{\#h} p_i = 1, \sum_{j=1}^{\#g} q_j = 1$, the sum of probabilities of membership and the sum of probabilities of non-membership is unity, i.e., the input PDHFEs have complete probabilistic information.

Remark 5. The fact that the original values were repeated indicates that the identical values were given by two different expert groups, and the probability distribution of those values may not be distributed equally, which supports the suggested method.

Example 1. Let $h_1 = \langle \begin{Bmatrix} 0.4|0.2,0.3|0.1 \\ 0.2|0.2,0.3|0.5 \end{Bmatrix}, \{0.4|1\} \rangle$ and $h_2 = \langle \begin{matrix} \{0.5|0.5,0.6|0.5\}, \\ \{0.3|0.6,0.4|0.4\} \end{matrix} \rangle$ be two PDHFEs.

According to *Algorithm 1*, the process of normalization can be carried out in the following manner:

Step 1. Given that $p_1^1 = 0.2$ and $p_2^1 = 0.5$, then $p_1^1 < p_2^1$, $\gamma_1^1(p_1^1) = \gamma_1^1(p_1^1) = 0.4|0.2$ and $\gamma_2^1(p_2^1) = \gamma_2^1(p_1^1) = 0.5|0.5$.

Step 2. Given that $p_1^1 < p_2^1$ and $(p_2^1) - (p_1^1) > (p_1^2)$, then $\gamma_1^2(p_1^2) = \gamma_1^2(p_1^2) = 0.3|0.1$ and $\gamma_2^2(p_2^2) = \gamma_2^1(p_1^2) = 0.5|0.1$.

Step 3. Given that $p_1^1 < p_2^1, p_2^1 - p_1^1 > p_1^2$ and $p_2^1 - p_1^1 + p_1^2 > p_1^3$, then $\gamma_1^3(p_1^3) = \gamma_1^3(p_1^3) = 0.2|0.2$ and $\gamma_2^3(p_2^3) = \gamma_2^1(p_1^3) = 0.5|0.2$.

Proceeding further for both set of membership and non-membership we will get the following NPDHFEs.

$$h_1 = \left\langle \begin{matrix} \begin{Bmatrix} 0.4|0.1,0.4|0.1,0.3|0.1,0.2|0.1 \\ 0.2|0.1,0.3|0.1,0.3|0.1,0.3|0.3 \end{Bmatrix}, \\ \begin{Bmatrix} 0.4|0.3,0.4|0.1,0.4|0.1,0.4|0.1, \\ 0.4|0.1,0.4|0.2,0.4|0.1 \end{Bmatrix} \end{matrix} \right\rangle \text{ and } h_2 = \begin{matrix} \begin{Bmatrix} 0.5|0.1,0.5|0.1,0.5|0.1,0.5|0.1 \\ 0.5|0.1,0.6|0.1,0.6|0.1,0.6|0.3 \end{Bmatrix}, \\ \begin{Bmatrix} 0.3|0.3,0.3|0.1,0.3|0.1,0.3|0.1, \\ 0.4|0.1,0.4|0.2,0.4|0.1 \end{Bmatrix}. \end{matrix}$$

3.2 | New Distance and Similarity Measures for PDHFS

Distance and similarity measures are the key concepts of decision-making. Many distance measures have been defined, among which Hamming distance, Euclidean distance, and Hausdorff distance are widely used. In fuzzy set theory, the distance measure was first proposed by [32]. Later, several distance and similarity measures for different fuzzy environments, such as IFS [33], [34], IVIFS, HFS [35], and DHFS [36], have been defined. These measures have many applications, such as in the field of pattern recognition, site selection, portfolio selection [31–33], decision-making [37–39], and renewable energy systems [40–42] As

aforementioned, PDHFS is quite excellent in expressing ambiguity and uncertainty; therefore, in this section, we have defined a series of novel distance and similarity measures that are helpful in finding the distance between two PDHFSs.

Let A and B be two PDHFSs, and let $\#h$, and $\#g$ be the total number of elements in the set of membership and non-membership, respectively. For sets A and B, $h_{A_{x_k}}^{\sigma(i)}$ and $h_{B_{x_k}}^{\sigma(i)}$is the i^{th} highest element of membership set and $p_{A_{x_k}}$ and $p_{B_{x_k}}$is their respective probability in sets A and B, respectively. Similarly, $g_{A_{x_k}}^{\sigma(j)}$ and $g_{B_{x_k}}^{\sigma(j)}$ is the j^{th} highest element of the non-membership set and $q_{A_{x_k}}$ and $q_{B_{x_k}}$ is their respective probability in sets A and B, respectively.

The distance between these two sets is given by:

Normalized Hamming distance measure

$$d(A,B)=\sum_{k=1}^{n}\frac{1}{n}\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}\right].$$

Normalized Euclidean distance measure

$$d(A,B)=\left[\sum_{k=1}^{n}\frac{1}{n}\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2}{2}\right]\right]^{1/2}.$$

Generalized normalized distance

$$d(A,B)=\left[\sum_{k=1}^{n}\frac{1}{n}\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^\lambda}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^\lambda}{2}\right]\right]^{1/\lambda},$$

where $\lambda > 0$.

Normalized Hamming Hausdorff distance

$$d(A,B)=\frac{1}{n}\sum_{k=1}^{n}\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}.$$

Normalized Euclidean Hausdorff distance

$$d(A,B)=\left[\frac{1}{n}\sum_{k=1}^{n}\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2\right\}\right]^{1/2}.$$

Generalized normalized Hausdorff distance

$$d(A,B)=\left[\frac{1}{n}\sum_{k=1}^{n}\max\left\{\max_{i}\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda},\max_{j}\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}\right\}\right]^{1/\lambda},$$

where $\lambda > 0$.

By combining the two normalized Hamming, Euclidean, and Hausdorff distances, a hybrid normalized PDHF distance, hybrid normalized PDHF Euclidean distance, and a generalized hybrid normalized PDHF distance is defined as follows:

Hybrid normalized Hamming distance measure

$$d(A,B)=\frac{1}{2n}\sum_{k=1}^{n}\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}+\max\left\{\max_{i}\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_{j}\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}\right].$$

Hybrid normalized Euclidean distance measure

$$d(A,B)=\left[\frac{1}{2n}\sum_{k=1}^{n}\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}}{2}+\max\left\{\max_{i}\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2},\max_{j}\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}\right\}\right]\right]^{1/2}.$$

Hybrid generalized normalized distance measure

$$d(A,B)=\left[\frac{1}{2n}\sum_{k=1}^{n}\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2}+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2},\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}\right\}\right]^{1/\lambda}.$$

Since each x_k perform a distinct role throughout the set X, therefore it should be weighted differently. So, we need to incorporate some weighted versions of the aforesaid distance measurements. Suppose the weight of the elements x_k are $w_k(k=1,2,\ldots n)$, with $w_k\epsilon[0,1]$ and $\sum_{k=1}^{n}w_k=1$. The weighted hamming distance, weighted Euclidean distance, and generalized weighted distance are defined as follows:

Weighted Hamming distance measure

$$d(A,B)=\sum_{k=1}^{n}w_k\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}\right].$$

Weighted Euclidean distance measure

$$d(A,B)=\left[\sum_{k=1}^{n}w_k\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}}{2}\right]\right]^{1/2}.$$

Generalized weighted distance measure

$$d(A,B)=\left[\sum_{k=1}^{n}w_k\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2}\right]\right]^{\frac{1}{\lambda}},$$

where $\lambda>0$.

Generalized weighted Hausdorff distance measure

$$d(A,B)=\left[\sum_{k=1}^{n} w_k \max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda},\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}\right\}\right]^{\frac{1}{\lambda}},$$

where $\lambda > 0$.

Weighted Hamming Hausdorff distance measure

$$d(A,B)=\sum_{k=1}^{n} w_k \max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}.$$

Weighted Euclidean Hausdorff distance measure

$$d(A,B)=\left[\sum_{k=1}^{n} w_k \max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2},\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}\right\}\right]^{\frac{1}{2}}.$$

Hybrid Weighted Hamming distance measure

$$d(A,B)=\sum_{k=1}^{n} w_k\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}\right].$$

Hybrid weighted Euclidean distance measure

$$d(A,B)=\left[\sum_{k=1}^{n} w_k\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2}{2}+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2},\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}\right\}\right]\right]^{1/2}.$$

Hybrid Generalized weighted distance measure

$$d(A,B)=\left[\sum_{k=1}^{n} w_k\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2}\right.\right.$$
$$+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2}$$
$$+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda},\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)\right.\right.$$
$$\left.\left.\left.\left.-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}\right\}\right]\right]^{1/\lambda}.$$

In the aforementioned distance measures, the variable x_k is random. If both $x\in X=[a,b]$ and the weight $w(x)$ of x obeying $w(x)\in[0,1]$ and $\int_a^b w(x)=1$ are continuous in X, then the distance measures defined above can be extended to a continuous fuzzy environment.

The continuous probabilistic dual hesitant weighted Hamming distance is defined as follows:

Continuous weighted Hamming distance measure

$$d(A,B)=\int_a^b w_x\left(\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}\right.$$
$$\left.+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}\right)dx.$$

Continuous weighted Euclidean distance measure

$$d(A,B)=\left[\int_a^b w_x\left(\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2}}{2}\right.\right.\right.$$
$$\left.\left.\left.+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}}{2}\right]\right)dx\right]^{\frac{1}{2}}.$$

Continuous weighted generalized distance measure

$$d(A,B)=\left[\int_a^b w_x\left(\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2}\right.\right.$$
$$\left.\left.+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2}\right)dx\right]^{\frac{1}{\lambda}},$$

where $\lambda>0$.

Continuous weighted Hamming Hausdorff distance measure

$$d(A,B)=\int_a^b w_x\left(\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}\right)dx.$$

Continuous weighted Euclidean Hausdorff distance measure

$$d(A,B)=\left[\int_a^b w_x\left\{\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2\right\}\right\}\right]^{\frac{1}{2}}dx.$$

Continuous weighted generalized Hausdorff distance measure

$$d(A,B)=\left[\int_a^b w_x\left\{\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^\lambda,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^\lambda\right\}\right\}\right]^{\frac{1}{\lambda}}dx,$$

where $\lambda > 0$.

Continuous weighted hybrid Hamming distance measure

$$\begin{aligned}d(A,B)=\int_a^b w_x\Bigg\{&\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}\\&+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}\\&+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}\Bigg\}dx.\end{aligned}$$

Continuous weighted hybrid Euclidean distance measure

$$\begin{aligned}d(A,B)=\Bigg[\int_a^b w_x\Bigg\{&\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2}{2}\\&+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2}{2}\\&+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2\right\}\Bigg\}dx\Bigg]^{\frac{1}{2}}.\end{aligned}$$

Continuous weighted hybrid generalized distance measure

$$d(A,B)=\left[\int_a^b w_x\left\{\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2}+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda},\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}\right\}\right\}dx\right]^{\frac{1}{\lambda}},$$

where $\lambda > 0$.

Continuous normalized Hamming distance measure

$$d(A,B)=\int_a^b\frac{1}{b-a}\left(\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}\right)dx.$$

Continuous normalized Euclidean distance measure

$$d(A,B)=\left[\int_a^b\frac{1}{b-a}\left(\left[\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{2}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{2}}{2}\right]\right)dx\right]^{\frac{1}{2}}.$$

Continuous normalized generalized distance measure

$$d(A,B)=\left[\int_a^b\frac{1}{b-a}\left(\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2}\right)dx\right]^{\frac{1}{\lambda}},$$

where $\lambda > 0$.

Continuous normalized Hamming Hausdorff distance measure

$$d(A,B)=\int_a^b \frac{1}{b-a}\left(\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}\right)dx.$$

Continuous normalized Euclidean Hausdorff distance measure

$$d(A,B)=\left[\int_a^b \frac{1}{b-a}\left\{\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2\right\}\right\}\right]^{\frac{1}{2}}dx.$$

Continuous normalized generalized Hausdorff distance measure

$$d(A,B)=\left[\int_a^b \frac{1}{b-a}\left\{\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^\lambda,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^\lambda\right\}\right\}\right]^{\frac{1}{\lambda}}dx,$$

where $\lambda > 0$.

Continuous normalized hybrid Hamming distance measure

$$d(A,B)=\int_a^b \frac{1}{b-a}\left\{\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|}{2}+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|\right\}\right\}dx.$$

Continuous normalized hybrid Euclidean distance measure

$$d(A,B)=\left[\int_a^b \frac{1}{b-a}\left\{\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2}{2}+\frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2}{2}+\max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)-h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^2,\max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)-g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^2\right\}\right\}dx\right]^{\frac{1}{2}}.$$

Continuous normalized hybrid generalized distance measure

$$d(A,B) = \left[\int_a^b \frac{1}{b-a}\left\{\frac{1}{\#h}\sum_{i=1}^{\#h}\frac{\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}}{2} + \frac{1}{\#g}\sum_{j=1}^{\#g}\frac{\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}}{2} + \max\left\{\max_i\left|h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right|^{\lambda}, \max_j\left|g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right|^{\lambda}\right\}\right\} dx\right]^{\frac{1}{\lambda}},$$

where $\lambda > 0$.

The proposed distance measurements must comply with the axiomatic definition given by:

Theorem 1. Let A and B be two PDHFSs, then the distance measures $d(A,B)$ satisfies the following conditions:

(P_1) $0 \leq d\,(A,B) \leq 1$,
(P_2) $d\,(A,B) = d\,(B,A)$,
(P_3) $d\,(A,B) = 0$ if $A = B$,
(P_4) If $A \subseteq B \subseteq C$, then $d(A,B) \leq d(A,C)$ and $d(B,C) \leq d(A,C)$.

Proof: let $X = \{x_1, x_2, x_3, \dots, x_n\}$ be the universal set A, B be two PDHFSs defined over X. Then, for each x_k, $k = 1,2,\dots,n$, we have

(P_1) since $0 \leq h_{A_{x_k}}^{\sigma(i)} \leq 1$ and $0 \leq p_{A_{x_k}} \leq 1$ for all $i = 1,2\dots,\#h$.

$$\Rightarrow 0 \leq h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) \leq 1$$

Similarly, $0 \leq h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right) \leq 1$ **(6)**

$$\Rightarrow 0 \leq h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right) \leq 1$$

Further,

$$\sum_{i=1}^{\#h} 0 \leq \sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right) \leq \sum_{i=1}^{\#h} 1.$$

Which leads to,

$$0 \leq \sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right) \leq \#h$$

$$\Rightarrow 0 \leq \frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right) \leq 1.$$

Thus, for any $\lambda > 0$, we have

$$0 \le \left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} \le 1.$$

Similarly, for non-membership $j = 1, 2, \ldots, \#g$

$$0 \le \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} \le 1.$$

Thus, adding these, we get,

$$0 \le \left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda} \le 1 + 1$$

$$\Rightarrow 0 \le \frac{1}{2}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right] \le 1.$$

Further,

$$\sum_{k=1}^{n} 0 \le \sum_{k=1}^{n}\frac{1}{2}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right] \le \sum_{k=1}^{n} 1$$

$$\Rightarrow 0 \le \sum_{k=1}^{n}\frac{1}{2}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right] \le n$$

$$\Rightarrow 0 \le \left[\sum_{k=1}^{n}\frac{1}{2n}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right]\right]^{1/\lambda} \le 1$$

$$\Rightarrow 0 \le d(A, B) \le 1.$$

(P_2)

$d(A,B) = \left[\sum_{k=1}^{n}\frac{1}{2n}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right]\right]^{1/\lambda} = \left[\sum_{k=1}^{n}\frac{1}{2n}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right) - h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right) - g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)\right)\right|^{\lambda}\right]\right]^{1/\lambda} = d(B,A).$

(P_3) for $A = B$, $h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) = h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)$ and $g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) = g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)$ therefore $\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{B_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{A_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right) = 0$ and $\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right) - g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right)\right) = 0.$

Therefore,

$$\Rightarrow \left[\sum_{k=1}^{n}\frac{1}{2n}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right]\right]^{1/\lambda} = 0.$$

$\Rightarrow d(A,B) = 0.$

The only if part, $d(A,B) = 0.$

$$\left[\sum_{k=1}^{n}\frac{1}{2n}\left[\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} + \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda}\right]\right]^{1/\lambda} = 0$$

$\Rightarrow h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) = h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)$ and $g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) = g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)$

$\Rightarrow A = B.$

(P_4) Since $A \subseteq B \subseteq C$, then $h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) \le h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right) \le h_{C_{x_k}}^{\sigma(i)}\left(p_{C_{x_k}}\right)$ and $g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) \ge g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right) \ge g_{C_{x_k}}^{\sigma(j)}\left(q_{C_{x_k}}\right)$ and hence, $\left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{B_{x_k}}^{\sigma(i)}\left(p_{B_{x_k}}\right)\right)\right|^{\lambda} \le \left|\frac{1}{\#h}\sum_{i=1}^{\#h}\left(h_{A_{x_k}}^{\sigma(i)}\left(p_{A_{x_k}}\right) - h_{C_{x_k}}^{\sigma(i)}\left(p_{C_{x_k}}\right)\right)\right|^{\lambda}$ and $\left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{B_{x_k}}^{\sigma(j)}\left(q_{B_{x_k}}\right)\right)\right|^{\lambda} \ge \left|\frac{1}{\#g}\sum_{j=1}^{\#g}\left(g_{A_{x_k}}^{\sigma(j)}\left(q_{A_{x_k}}\right) - g_{C_{x_k}}^{\sigma(j)}\left(q_{C_{x_k}}\right)\right)\right|^{\lambda}.$

Therefore $d(A,B) \le d(A,C).$

With respect to the corresponding similarity measures, they can be easily obtained by utilizing the formula $s(A, B) = 1 - d(A, B)$.

Theorem 2. Let A and B be two PDHFSs, then the similarity measures $s(A, B)$ satisfies the following conditions:

(P_1) $0 \leq s(A, B) \leq 1$.
(P_2) $s(A, B) = s(B, A)$.
(P_3) $s(A, B) = 0$ if $A = B$.

Like the other fuzzy sets, the relationship between $s(A, B)$ and $d(A, B)$ also obeys the formulas that $s(A, B) = 1 - d(A, B)$. Therefore, the axioms for similarity measures can easily be proven based on the distance measures defined above.

4 | PDHF-ORESTE Approach

The ORESTE method is an outranking approach that does not need the crisp criterion weights. Furthermore, it may demonstrate precise differences between alternatives in terms of the preference relation, indifference relation, and incomparability relation. Since it is an effective ranking method, it has been developed within fuzzy environments to address a range of evaluation data [43–45].

4.1 | Problem Description

For a decision-making problem, let $A = \{A_1, A_2 \dots A_m\}$ be the set of alternatives, and $C = \{C_1, C_2 \dots C_n\}$ be the set of criteria, and $\upsilon = \{\upsilon_1, \upsilon_2, \dots \upsilon_n\}$ be criteria's weight set; Let $D = \{d_1, d_2 \dots d_z\}$ be decision-makers set and $w = \{w_1, w_2 \dots w_z\}$ be DM's weight set. Accordingly, the criteria's weight is given by PDHFEs $h_j = \{\gamma_j^s(p_j^s), \eta_j^t(g_j^t) | s = 1,2, \dots, \#h_j, t = 1,2, \dots, \#g_j\}$, $j = 1,2, \dots, n$. Each DM $d_k, k = 1,2, \dots, z$ provide their evaluation of alternatives A_i with respect to criterion C_j. The evaluation is given in the form of PDHFEs.

$$h_{ij,k} = \{\gamma_{ij,k}^s(p_{ij,k}^s), \eta_{ij,k}^t(g_{ij,k}^t) | s = 1,2, \dots, \#h_{ij,k}, t = 1,2, \dots, \#g_{ij,k}\}. \tag{7}$$

The PDHFE judgement matrix $I_k = (I_{ij})_{m\times n}$ of DM d_k, $k = 1,2, \dots, z$ is given by,

$$\begin{array}{cccccc} & C_1 & \cdots & C_j & \cdots & C_n \\ A_1 & h_{11,k} & \cdots & h_{1j,k} & \cdots & h_{1n,k} \\ \vdots & \vdots & & \vdots & & \vdots \\ A_i & h_{i1,k} & \cdots & h_{ij,k} & \cdots & h_{in,k} \\ \vdots & \vdots & & \vdots & & \vdots \\ A_m & h_{m1,k} & \cdots & h_{mj,k} & \cdots & h_{mn,k} \end{array}$$

4.2 | ORESTE Approach for PDHFEs

Being an excellent ranking approach, OTRESTE is used to solve numerous MCDM problems in different fuzzy environments, such as hesitant fuzzy information and intuitionistic, probabilistic, and PHF environments. In this section, the ORESTE approach for PDHFEs [46] has been discussed. It consists of two stages, namely, deriving the Weak Ranking (WR) and second deriving the Strong Ranking (SR) of the alternatives. The flow chart of the ORESTE approach for PDHFEs is represented in *Fig. 1*.

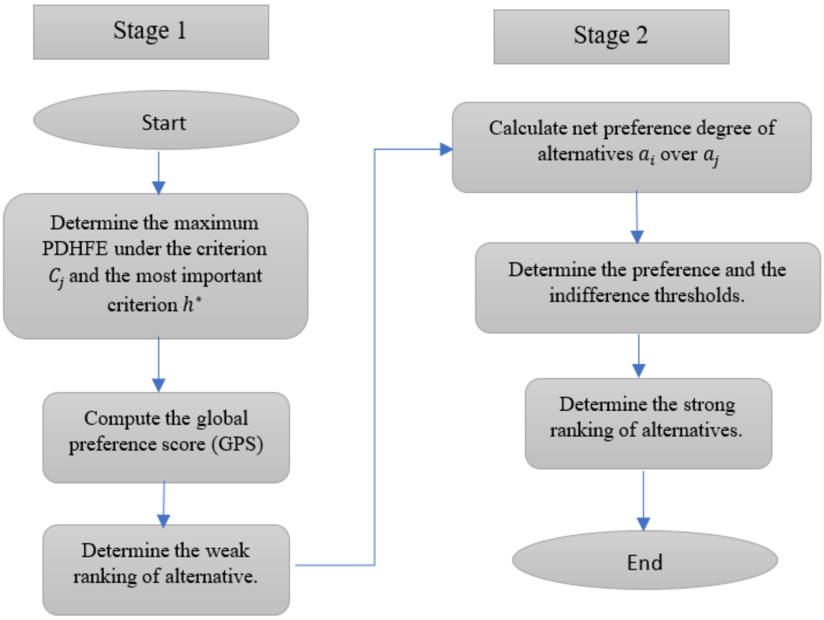

Fig. 1. ORESTE approach for PDHFEs.

4.3 | Algorithm for Group Decision Making with NPDHFEs

This section contains an approach to solving MCDM problems by using PDHFEs. The flow chart of the algorithm for group decision-making with PDHFEs is represented in *Fig. 2*. The primary technique is outlined

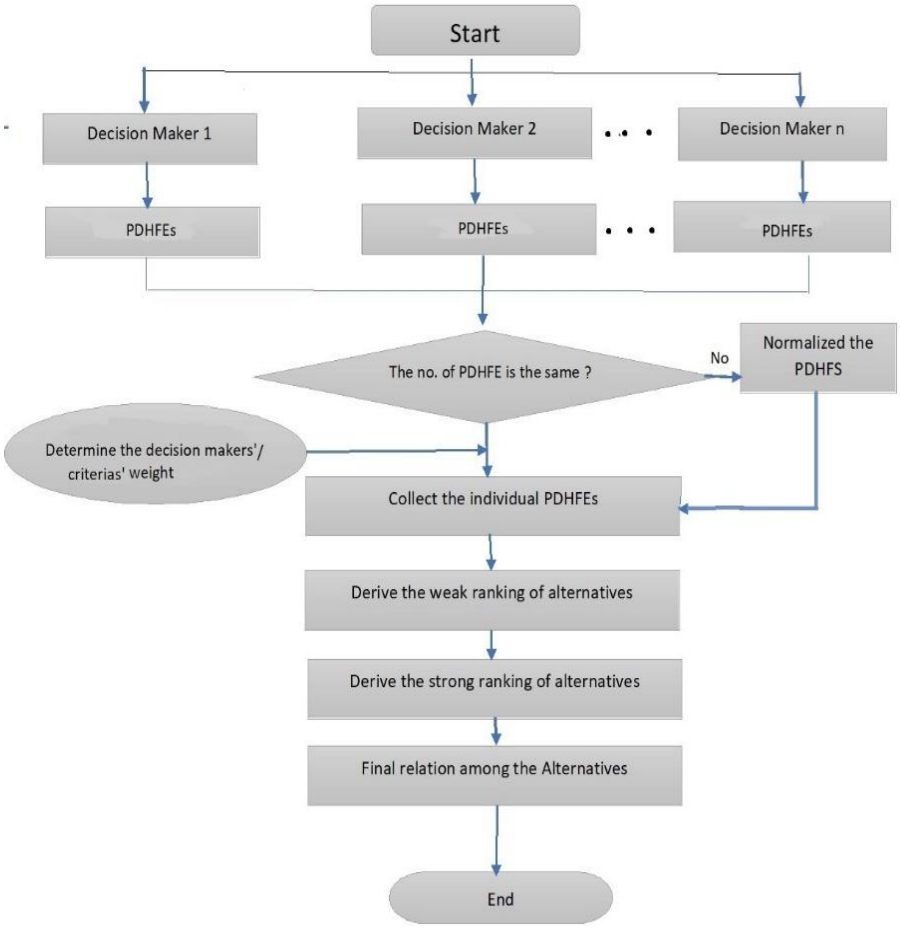

Fig. 2. Group decision making for PDHFEs.

5 | Group Decision-Making with PDHFEs

In recent years, several studies have been conducted on energy optimization, focusing on the fields of medical diagnosis, pattern recognition, site selection, and decision-making. MCDM and MCGDM are widely utilized in energy planning research. Several MCDM techniques have been developed over the years. For an energy company, this study aims to develop a methodology to analyze renewable energy plants. The fuzzy ORESTE [45] technique has been used to rank renewable energy plants based on the perspectives of stakeholders from various categories.

5.1 | Illustrative Example 1 (Renewable energy development plant)

Installing renewable energy plants benefits the local community in economic, social, and environmental ways. Local communities may not usually support investments in renewable energy projects. When making an investment decision, several factors are taken into account. These factors include the local community's adaptation and support.

Several criteria are considered while installing renewable energy plants and their adoption by local communities. These criteria may include a contribution to the region (C_1), the generation of business opportunities (C_2), and the environmental impact (C_3). All these three criteria are benefit criteria. In the light of the three determined criteria, wind (A_1), solar (A_2), hydraulic (A_3), and geothermal (A_4) energy types have been evaluated. These energies have been evaluated by referring to the views of stakeholders from different groups, such as environmental organizations, local communities, and authorities from leading institutes in the energy market. The fuzzy ORESTE method has been applied to rank the renewable energy plants. The fuzzy ORESTE method aims to obtain the most suitable renewable energy plant with respect to the given criteria. The fuzzy ORESTE method has been discussed as follows:

Step 1. The stakeholders' evaluations are collected.

The stakeholders have collected the individual PDHFEs, and the decision matrix (*Table 1*) has been constructed.

Table 1. The PDHF decision information matrix R.

	C_1	C_2	C_3
A_1	$\langle\begin{Bmatrix}0.5\|0.2,0.3\|0.1\\0.6\|0.2,0.2\|0.5\end{Bmatrix},\{0.3\|1\}\rangle$	$\langle\begin{matrix}\{0.3\|0.5,0.1\|0.5\},\\\{0.4\|0.6,0.2\|0.4\}\end{matrix}\rangle$	$\langle\begin{matrix}\{0.2\|0.4,0.3\|0.6\},\\\{0.5\|1\}\end{matrix}\rangle$
A_2	$\langle\begin{matrix}\{0.6\|1\},\\\begin{Bmatrix}0.3\|0.3,0.2\|0.2,\\0.1\|0.4,0.3\|0.1\end{Bmatrix}\end{matrix}\rangle$	$\langle\begin{matrix}\{0.5\|0.4,0.4\|0.6\},\\\{0.1\|1\}\end{matrix}\rangle$	$\langle\begin{matrix}\begin{Bmatrix}0.2\|0.1,0.4\|0.1,\\0.3\|0.5,0.6\|0.3\end{Bmatrix},\\\{0.1\|0.4,0.4\|0.6\}\end{matrix}\rangle$
A_3	$\langle\begin{matrix}\{0.3\|0.4,0.5\|0.6\},\\\{0.3\|1\}\end{matrix}\rangle$	$\langle\begin{matrix}\{0.3\|0.6,0.6\|0.4\},\\\begin{Bmatrix}0.1\|0.5,0.4\|0.2,\\0.3\|0.2,0.2\|0.1\end{Bmatrix}\end{matrix}\rangle$	$\langle\begin{matrix}\{0.2\|0.5,0.4\|0.5\},\\\{0.4\|0.6,0.3\|0.4\}\end{matrix}\rangle$
A_4	$\langle\begin{matrix}\{0.1\|0.7,0.2\|0.3\},\\\{0.3\|0.4,0.7\|0.6\}\end{matrix}\rangle$	$\langle\begin{matrix}\{0.2\|0.3,0.5\|0.7\},\\\{0.2\|1\}\end{matrix}\rangle$	$\langle\begin{matrix}\{0.1\|0.6,0.2\|0.4\},\\\{0.8\|0.3,0.3\|0.7\}\end{matrix}\rangle$

Step 2. Calculate the individual NPDHFEs. With the help of *Algorithm 1*, NPDHFEs are obtained as follows (*Table 2*). To illustrate the normalization process, let us consider $h_{11} = \langle\{0.5|0.2,0.3|0.1,0.6|0.2,0.2|0.5\},\{0.3|1\}\rangle$ and $h_{12} = \langle\{0.3|0.5,0.1|0.5\},\{0.4|0.6,0.2|0.4\}\rangle$ be two PDHFEs.

The normalization process discussed in *Algorithm 1* has been illustrated in the following steps:

Step 2.1. $p_1^1 = 0.2$ and $p_2^1 = 0.5$, then $p_1^1 < p_2^1, \gamma_1^1(p_1^1) = \gamma_1^1(p_1^1) = 0.5|0.2$ and $\gamma_2^1(p_2^1) = \gamma_2^1(p_1^1) = 0.3|0.2$

Step 2.2. Determine the second element.

Given that $p_1^1 < p_2^1$ and $p_2^1 - p_1^1 > p_1^2$ then $\gamma_1^2(p_1^2) = \gamma_1^2(p_1^2) = 0.3|0.1$ and $\gamma_2^2(p_2^2) = \gamma_2^1(p_1^2) = 0.3|0.1$

Step 2.3. Determine the third element.

Given that $p_1^1 < p_2^1$ and $p_2^1 - p_1^1 > p_1^2$ and $p_2^1 - p_1^1 + p_1^2 > p_1^3$ then $\gamma_1^3(p_1^3) = \gamma_1^3(p_1^3) = 0.6|0.2$ and $\gamma_2^3(p_2^3) = \gamma_2^1(p_1^3) = 0.3|0.2$.

Further proceeding in the same way, we will get the following normalized PDHFEs.

$$h_{11}^N = \langle\{0.5|0.2, 0.3|0.1, 0.6|0.2, 0.2|0.5\}, \{0.3|0.6, 0.3|0.4\}\rangle.$$

and

$$h_{12}^N = \langle\{0.3|0.2, 0.3|0.1, 0.3|0.2, 0.1|0.5\}, \{0.4|0.6, 0.2|0.4\}\rangle.$$

Given that $p_1^1 + p_1^2 + p_1^3 + p_1^4 = 1$ and $p_2^1 + p_2^2 + p_2^3 + p_2^4 = 1$; similarly, we will proceed further.

Table 2. Collective NPDHF decision matrix.

	C_1	C_2	C_3
A_1	$\left\langle \begin{Bmatrix} 0.5\|0.1, 0.5\|0.1, 0.3\|0.1, 0.6\|0.1 \\ 0.6\|0.1, 0.2\|0.1, 0.2\|0.1, 0.2\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.3\|0.3, 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, \\ 0.3\|0.1, 0.3\|0.2, 0.3\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, 0.3\|0.1 \\ 0.3\|0.1, 0.1\|0.1, 0.1\|0.1, 0.1\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.4\|0.3, 0.4\|0.1, 0.4\|0.1, 0.4\|0.1, \\ 0.2\|0.1, 0.2\|0.2, 0.2\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.2\|0.1, 0.2\|0.1, 0.2\|0.1, 0.2\|0.1 \\ 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, 0.3\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.5\|0.3, 0.5\|0.1, 0.5\|0.1, 0.5\|0.1, \\ 0.5\|0.1, 0.5\|0.2, 0.5\|0.1 \end{Bmatrix} \right\rangle$
A_2	$\left\langle \begin{Bmatrix} 0.6\|0.1, 0.6\|0.1, 0.6\|0.1, 0.6\|0.1 \\ 0.6\|0.1, 0.6\|0.1, 0.6\|0.1, 0.6\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.3\|0.3, 0.2\|0.1, 0.2\|0.1, 0.1\|0.1, \\ 0.1\|0.1, 0.1\|0.2, 0.3\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.5\|0.1, 0.5\|0.1, 0.5\|0.1, 0.5\|0.1 \\ 0.4\|0.1, 0.4\|0.1, 0.4\|0.1, 0.4\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.1\|0.3, 0.1\|0.1, 0.1\|0.1, 0.1\|0.1, \\ 0.1\|0.1, 0.1\|0.2, 0.1\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.2\|0.1, 0.4\|0.1, 0.3\|0.1, 0.3\|0.1 \\ 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, 0.6\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.1\|0.3, 0.1\|0.1, 0.4\|0.1, 0.4\|0.1, \\ 0.4\|0.1, 0.4\|0.2, 0.4\|0.1 \end{Bmatrix} \right\rangle$
A_3	$\left\langle \begin{Bmatrix} 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, 0.3\|0.1 \\ 0.5\|0.1, 0.5\|0.1, 0.5\|0.1, 0.5\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.3\|0.3, 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, \\ 0.3\|0.1, 0.3\|0.2, 0.3\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, 0.3\|0.1 \\ 0.3\|0.1, 0.3\|0.1, 0.6\|0.1, 0.6\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.1\|0.3, 0.1\|0.1, 0.1\|0.1, 0.4\|0.1, \\ 0.4\|0.1, 0.3\|0.2, 0.2\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.2\|0.1, 0.2\|0.1, 0.2\|0.1, 0.2\|0.1 \\ 0.2\|0.1, 0.4\|0.1, 0.4\|0.1, 0.4\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.4\|0.3, 0.4\|0.1, 0.4\|0.1, 0.4\|0.1, \\ 0.3\|0.1, 0.3\|0.2, 0.3\|0.1 \end{Bmatrix} \right\rangle$
A_4	$\left\langle \begin{Bmatrix} 0.1\|0.1, 0.1\|0.1, 0.1\|0.1, 0.1\|0.1 \\ 0.1\|0.1, 0.1\|0.1, 0.1\|0.1, 0.2\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.3\|0.3, 0.3\|0.1, 0.7\|0.1, 0.7\|0.1, \\ 0.7\|0.1, 0.7\|0.2, 0.7\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.2\|0.1, 0.2\|0.1, 0.2\|0.1, 0.5\|0.1 \\ 0.5\|0.1, 0.5\|0.1, 0.5\|0.1, 0.5\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.2\|0.3, 0.2\|0.1, 0.2\|0.1, 0.2\|0.1, \\ 0.2\|0.1, 0.2\|0.2, 0.2\|0.1 \end{Bmatrix} \right\rangle$	$\left\langle \begin{Bmatrix} 0.1\|0.1, 0.1\|0.1, 0.1\|0.1, 0.1\|0.1 \\ 0.1\|0.1, 0.1\|0.1, 0.2\|0.1, 0.2\|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.8\|0.3, 0.3\|0.1, 0.3\|0.1, 0.3\|0.1, \\ 0.3\|0.1, 0.3\|0.2, 0.3\|0.1 \end{Bmatrix} \right\rangle$

Step 3. Let us consider the criteria weight to be {0.32, 0.41, 0.27}.

Step 4. Individual NPDHFEs are aggregated by using the following aggregation operator.

$$PDHFEWA = \bigcup_{\substack{\gamma_1 \epsilon h_1, \gamma_2 \epsilon h_2 \ldots \gamma_n \epsilon h_n \\ \eta_1 \epsilon g_1, \eta_2 \epsilon g_2 \ldots \eta_n \epsilon g_n}} \left\{ \left\{ (1 - \prod_{i=1}^{n} (1 - \gamma_i)^{\omega_i}) \middle| \prod_{i=1}^{n} p_{\gamma_i} \right\}, \left\{ \prod_{j=1}^{n} (\eta_j)^{\omega_j} \middle| \prod_{j=1}^{n} q_{\eta_j} \right\} \right\}.$$

Step 5. WR of the alternatives is calculated [46] and represented as follows.

$$h_1^* = \left\langle \begin{Bmatrix} 0.7|0.1, 0.7|0.1, 0.7|0.1, 0.7|0.1 \\ 0.7|0.1, 0.7|0.1, 0.7|0.1, 0.7|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.4|0.3, 0.3|0.1, 0.3|0.1, 0.2|0.1, \\ 0.2|0.1, 0.2|0.2, 0.4|0.1 \end{Bmatrix} \right\rangle.$$

$$h_2^* = \left\langle \begin{Bmatrix} 0.5|0.1, 0.5|0.1, 0.5|0.1, 0.5|0.1 \\ 0.5|0.1, 0.6|0.1, 0.6|0.1, 0.6|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.2|0.3, 0.2|0.1, 0.2|0.1, 0.2|0.1, \\ 0.2|0.1, 0.2|0.2, 0.2|0.1 \end{Bmatrix} \right\rangle.$$

$$h_3^* = \left\langle \begin{Bmatrix} 0.4|0.1, 0.4|0.1, 0.4|0.1, 0.4|0.1 \\ 0.4|0.1, 0.5|0.1, 0.5|0.1, 0.5|0.3 \end{Bmatrix}, \begin{Bmatrix} 0.3|0.3, 0.3|0.1, 0.2|0.1, 0.2|0.1 \\ 0.2|0.1, 0.2|0.2, 0.2|0.1| \end{Bmatrix} \right\rangle.$$

$$h^* = 0.415.$$

The Global preference score is obtained by

$$D(A_{ij}) = \sqrt{\mu(d_{ij})^2 + (1-\mu)(d_j)^2}.$$

For the value of parameter $\mu = 0.5$, the following GPS is obtained (*Table 3*).

Table 3. Global preference score.

	C_1	C_2	C_3
A_1	0.032234	0.050003	0.005608
A_2	0.03182	0.049531	0.001176
A_3	0.032091	0.049697	0.003222
A_4	0.034058	0.049695	0.006358

The average preference scores of the alternatives are calculated as follows:

$$D(A_1) = 0.029282, D(A_2) = 0.027509, D(A_3) = 0.028337, D(A_4) = 0.030037.$$

As it is clear by the average preference score $D(A_4) > D(A_1) > D(A_3) > D(A_2)$, WR of the alternatives would be $A_2 \succ A_3 \succ A_1 \succ A_4$.

Step 6. Obtain SR of the alternatives.

The net preference degree of the alternatives can be determined as follows:

$$\Delta\zeta(A_1, A_2) = -0.0017729, \Delta\zeta(A_1, A_3) = -0.000945184, \Delta\zeta(A_1, A_4) = 0.000755,$$
$$\Delta\zeta(A_2, A_3) = 0.000828, \Delta\zeta(A_2, A_4) = 0.002528125, \Delta\zeta(A_3, A_4) = 0.0017.$$

The threshold θ will be determined as $\theta \in [0,0.007]$.

Let $\theta = 0.001$, since s = 3, the indifference threshold ϑ is determined as $\vartheta = 8.33*10\text{-}5$.

As by the calculation $|\Delta\zeta(A_1, A_2)| > \theta$ and $\zeta(A_2, A_1) > 0$, then by [46], $A_2 \succ A_1$. Similarly, $A_3 \succ A_1, A_1 \succ A_4$, $A_2 \succ A_3, A_2 \succ A_4$ and $A_3 \succ A_4$ will be determined, so the SR of the alternatives is $A_2 \succ A_3 \succ A_1 \succ A_4$.

Hence, while opting for the best alternative among all four alternatives, the solar energy plant is most optimal to be installed in accordance with the given criteria, followed by hydraulic and wind as second and third, respectively, and geothermal is the least recommended energy plant.

5.2 | Illustrative Example 2 (SSS problem)

To validate the applicability of the developed approach, a numerical example of SSS [47] has been taken. In the illustration, there are four sustainable suppliers $A_i (i = 1,2,3,4)$ to be evaluated based on the following three attributes. Three attributes are: the economical factor (C_1), the social factor (C_2) and the environmental factor (C_3). The global preference score obtained is shown in *Table 4*.

The Global preference score is obtained by

$$D(A_{ij}) = \sqrt{\mu(d_{ij})^2 + (1-\mu)(d_j)^2}.$$

For the value of parameter $\mu = 0.5$, the following GPS is obtained (*Table 6*).

Stage 1. Obtain the WR of the alternatives by using the algorithm developed by Gupta et al. [45].

Table 4. Global preference score.

	C_1	C_2	C_3
A_1	0.066539547	0.026051472	0.017172593
A_2	0.06179881	0.02677768	0.003535534
A_3	0.065307054	0.02721355	0.012437503
A_4	0.06444026	0.035572881	0.017298862

The average preference scores of the alternatives are calculated as follows:

$D(A_1)$ =0.036588, $D(A_2)$= 0.030704, $D(A_3)$= 0.034986, $D(A_4)$= 0.039104.

As is clear by the average preference score $D(A_4) > D(A_1) > D(A_3) > D(A_2)$, WR of the alternatives would be $A_2 \succ A_3 \succ A_1 \succ A_4$.

Stage 2. Obtain a SR of the alternatives.

The net preference degree of the alternatives can be determined as follows:

$\Delta\zeta(A_1, A_2) = -0.00588$, $\Delta\zeta(A_1, A_3) = -0.001602$, $\Delta\zeta(A_1, A_4)$ =0.00251613,
$\Delta\zeta(A_2, A_3) = 0.00428$, $\Delta\zeta(A_2, A_4) = 0.008399$, $\Delta\zeta(A_3, A_4)$ =0.004117.

The threshold θ will be determined as $\theta \in [0,0.007]$.

Let θ = 0.001, since s = 3, the indifference threshold ϑ is determined as ϑ = 8.33*10-5.

As by the calculation $|\Delta\zeta(A_1, A_2)| > \theta$ and $\zeta(A_2, A_1) > 0$, then [46], $A_2 \succ A_1$. Similarly, $A_3 \succ A_1$, $A_1 \succ A_4$, $A_2 \succ A_3$, $A_2 \succ A_4$ and $A_3 \succ A_4$ will be determined, so the SR of the alternatives is $A_2 \succ A_3 \succ A_1 \succ A_4$.

5.2.1 | Result and discussion

In this section, results based on different values of parameter μ, preference threshold, θ, and indifference threshold ϑ have been discussed. Also, a comparative study has been conducted using previously defined methods to validate the applicability of the given approach in supplier selection.

5.2.1.1 | Result based on different values of μ

The WR of the alternatives is measured by average preference scores, while the average preference scores differ based on different values of μ (*Table 5*). After determining the net preference degree of the alternatives, preference threshold θ, and indifference threshold ϑ, the results of the SR of the alternatives based on different values of μ are presented in *Table 6*.

Since different values of μ have been discussed in *Table 5* and *Table 6*, although different values of μ generate different average preference scores and net preference degrees, the WR of the alternatives and the SR of the alternatives do not change (except at $\mu = 0$). Thus, while opting for the best alternative among all four alternatives, A_2 consistently rank first, followed by A_3 and A_1 as second and third, respectively, A_4 is the least recommended alternative. This results in the value of μ having no significant influence on orders of the alternatives.

Table 5. WR of the alternatives based on the different values of μ.

μ	$D(a_1)$	$D(a_2)$	$D(a_3)$	$D(a_4)$	Ranking of Average Preference Score	WR of the Alternatives
0	0.04086	0.04086	0.04086	0.04086	$D(A_1) = D(A_2) = D(A_3) = D(A_4)$	$A_1 \sim A_2 \sim A_3 \sim A_4$
0.2	0.04081	0.0375	0.0398	0.04177	$D(A_4) > D(A_1) > D(A_3) > D(A_2)$	$A_2 \succ A_3 \succ A_1 \succ A_4$
0.4	0.03823	0.03317	0.0368	0.0402	$D(A_4) > D(A_1) > D(A_3) > D(A_2)$	$A_2 \succ A_3 \succ A_1 \succ A_4$
0.6	0.03471	0.02797	0.3295	0.03779	$D(A_4) > D(A_1) > D(A_3) > D(A_2)$	$A_2 \succ A_3 \succ A_1 \succ A_4$
0.8	0.03008	0.02122	0.02803	0.03437	$D(A_4) > D(A_1) > D(A_3) > D(A_2)$	$A_2 \succ A_3 \succ A_1 \succ A_4$
1	0.02333	0.00723	0.02101	0.02878	$D(A_4) > D(A_1) > D(A_3) > D(A_2)$	$A_2 \succ A_3 \succ A_1 \succ A_4$

Table 6. SR of the alternatives based on different values of μ.

μ	θ	ϑ	$\Delta\rho(A_1, A_2)$	$\Delta\rho(A_1, A_3)$	$\Delta\rho(A_1, A_4)$	$\Delta\rho(A_2, A_3)$	$\Delta\rho(A_2, A_4)$	$\Delta\rho(A_3, A_4)$	WR of the alternatives
0	0.000	0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	$A_1 \sim A_2 \sim A_3 \sim A_4$
0.2	0.001	0.000833	-0.003309064	-0.001005935	0.000958535	0.002303129	0.004267599	0.00196447	$A_2 \succ A_3 \succ A_1 \succ A_4$
0.4	0.001	0.000833	-0.005053966	-0.001429147	0.001975518	0.003624819	0.007029484	0.003404666	$A_2 \succ A_3 \succ A_1 \succ A_4$
0.6	0.001	0.000833	-0.006746434	-0.001759754	0.003081581	0.00498668	0.009828015	0.004841334	$A_2 \succ A_3 \succ A_1 \succ A_4$
0.8	0.002	0.001667	-0.008855781	-0.002048154	0.004290106	0.006807627	0.013145887	0.00633826	$A_2 \succ A_3 \succ A_1 \succ A_4$
1	0.002	0.001667	-0.01610119	-0.002321429	0.005446429	0.013779762	0.021547619	0.007767857	$A_2 \succ A_3 \succ A_1 \succ A_4$

5.2.2 | Sensitive Analysis

The ranking of alternatives for different proposed distance measures has been obtained. Although different values of λ generate different distance measures, the ranking of the alternatives does not change. Thus, while opting for the best alternative among all four alternatives, A_2 consistently rank first, followed by A_3 and A_1 as second and third, respectively, A_4 is the least recommended alternative. As it is clear from *Table 7*, for higher values of λ, the ranking of the alternatives has been changed except for the optimal alternative. Therefore,

the obtained distance measures are capable of obtaining the same optimal solution even for different values of λ, which shows the outperformance of the proposed measure.

Table 7. Ranking for different distance measures.

For Different Distance Measures	Weak Ranking	Strong Ranking
$\lambda = 1$	$A_2 \succ A_3 \succ A_1 \succ A_4$	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\lambda = 2$	$A_2 \succ A_3 \succ A_1 \succ A_4$	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\lambda = 5$	$A_2 \succ A_1 \succ A_3 \succ A_4$	$A_2 \succ A_1 \succ A_3 \succ A_4$
$\lambda = 10$	$A_2 \succ A_1 \succ A_3 \succ A_4$	$A_2 \succ A_1 \succ A_3 \succ A_4$

5.2.3 | Comparative study with other approaches

To validate the practicality of the proposed technique, comparative research with different methodologies has been done by considering the illustrative *Example 2* discussed in Section 5.2. Several decision-making approaches based on different aggregation operators have been discussed in *Table 8*. According to the ranking obtained for suppliers' selection by different methods given in *Table 8* and *Fig. 3*, the ranking of the four alternatives is slightly different for different methods. However, the best and worst alternatives are the same for different methods. *Table 8* shows that the ranking results obtained by different aggregation operators are identical to the proposed approaches, demonstrating that the developed method is effective.

Table 8. Ranking of alternatives obtained by different aggregation operators.

S. No.	Method	Ranking
1	WPDHFMSMA operator [48]	$A_2 \succ A_3 \succ A_1 \succ A_4$
2	PDHFWEA operator [46]	$A_2 \succ A_3 \succ A_1 \succ A_4$
3	PDHFWEG operator [46]	$A_2 \succ A_3 \succ A_1 \succ A_4$
4	Traditional PDHF TODIM method [49]	$A_2 \succ A_1 \succ A_3 \succ A_4$
5	PDHF VIKOR method [50]	$A_2 \succ A_3 \succ A_4 \succ A_1$
6	PDHFWPGMSM operator [48]	$A_2 \succ A_4 \succ A_3 \succ A_1$
7	Proposed ORESTE approach	$A_2 \succ A_3 \succ A_1 \succ A_4$

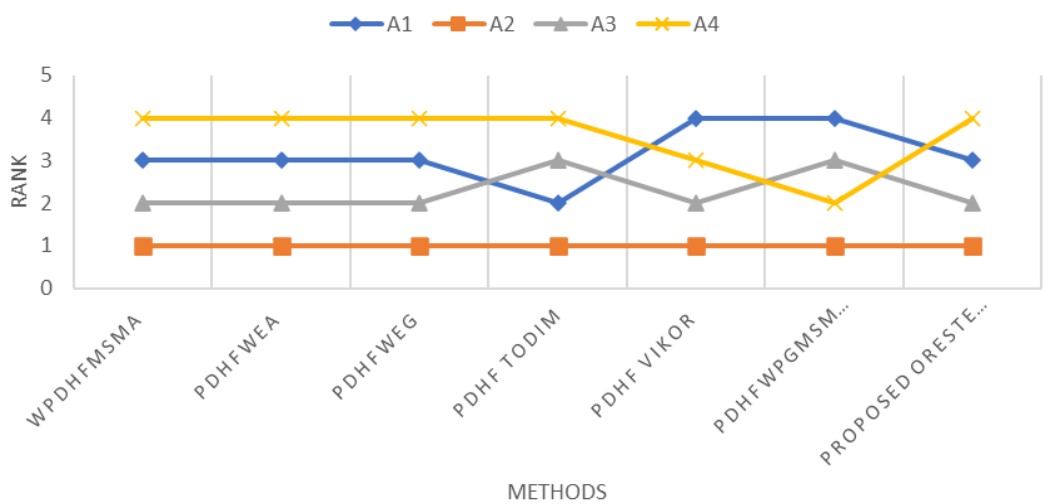

Fig. 3. The ranking of the four Sustainable Suppliers obtained by several methods.

6 | Conclusion

The energy requirement is increasing with the increase in population day by day. On the other hand, available energy sources are not enough to meet energy demand in day-to-day life. Therefore, renewable energy sources are essential to tackle climate change and can be seen as an alternative in terms of being clean and

environmentally sensitive. As an important extension of fuzzy numbers, PDHFEs are more flexible in handling uncertainty in the data and capturing information with some more degree of freedom.

In this paper, the French Organization Rangement Et Synthese De Ronnees Relationnelles' (ORESTE) approach for MCDM technique has been developed to rank renewable energy plants with probabilistic dual hesitant fuzzy information. ORESTE is an excellent ranking technique that provides preference relations among the alternatives. First, a series of new distance and similarity measures were developed in both discrete and continuous environments. Second, a new normalization process was developed to obtain the normalized NPDHFEs. To illustrate the effectiveness of the developed method, a sustainable energy development problem has been identified. To validate the effectiveness of the developed method, the Sustainable Supplier Selection (SSS) problem has also been taken as an example. A comparative study has also been discussed to define the supremacy of proposed over previously defined methods along with sensitive analysis.

The decision-making framework of the proposed PDHF ORESTE method makes decision results more reasonable and accurate than other MCDM methods because PDHF information can deal with uncertainty and fuzziness, which can be useful to ensure the integrity and accuracy of the developed method. The only limitation of the proposed method may be due to the hesitant bi-fuzzy data, in which there is no restriction in the domain.

In future work, our goal will be to develop more decision-making methods under the PDHF environment and apply the ORESTE approach to different fields like medical diagnosis, engineering management, and stock selection problems. In future work, we may define the ORESTE approach under a hesitant bi-fuzzy environment.

Author Contribution

Soniya Gupta, Dheeraj Kumar Joshi, Natasha Awasthi: Investigation, Resources, Validation, Visualization, Writing - review & editing.

Dheeraj Kumar Joshi and Soniya Gupta: Writing - original draft, Supervision, Visualization.

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Data Availability

The data is provided in the manuscript.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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